J. Fluid Mech. (2009), *vol.* 619, *pp.* 277–293. © 2008 Cambridge University Press
doi:10.1017/S0022112008004485 Printed in the United Kingdom

Galerkin representations and fundamental solutions for an axisymmetric microstretch fluid flow

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(Received 25 February 2008 and in revised form 3 October 2008)

The method of associated matrices is used to obtain Galerkin type representations. Fundamental solutions are then obtained for the cases of a point body couple and a point microstretch force. A formula for calculating the total couple acting on a rigid body rotating axi-symmetrically in a microstretch fluid is deduced. A generalized reciprocal theorem is deduced. An application for a rigid sphere rotating in a microstretch fluid is discussed. The results of the application are represented graphically.

1. Introduction

The theory of microfluids presented by Eringen in 1964 gives a mathematical formulation of a general theory of fluid microcontinua. It presents a new balance law, namely the law of conservation of micro-inertia. Eringen (1964) adapted a physical model in which each continuum particle is assigned a substructure; i.e. each material volume element contains microvolume elements which can translate, rotate and deform independent of the motion of the macrovolume. However, each deformation of a macrovolume element can be expected to produce a subsequent deformation of the microvolume elements.

A subclass of the microfluids is that of the microstretch fluids (sometimes called Eringen fluids). In this type of fluids, material points are considered to stretch, expand or contract, in addition to rotating about their centroids. The microstretch fluids have seven degrees of freedom: three for translation, three for rotation and one for stretch. These fluids model slurries, paper pulps, colloidal fluids, animal blood, bubbly fluids and other biological fluids (Eringen 1998). Numerous experiments have been performed during the last two decades that show that some of these fluids exhibit microstretch effect (see Rogausch 1976; Nagasawa 1981; Bird, Armstrong & Hassager 1987; Kröger 2004; Bor-Kucukatay *et al.* 2005 and the references therein).

Blood consists of suspensions of particulate cells in plasma of organic and inorganic substances. Red blood makes up 99 % of the volume of particulate matter (Eringen 1998). The volumetric percentage of red cells in blood varies between 40 % and 50 % of the whole blood. Human red blood cells are biconcave, disclike particles with an average diameter of 7.6 μm and a thickness of 2.8 μm. They are highly flexible. The shear modulus of red blood cells is about 0.004 dyn cm^{-1}. White blood cells in blood are nearly spherical and about 8 μm in diameter. They are stiffer but much less numerous than red cells. Blood also contains a small amount of platelets in the form of ellipsoidal particles. It seems that human blood (and for that matter animal blood as well) can be modelled with microstretch fluid continuum as a suspension rheology

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with flexible suspensions in flowing through small arteries (Eringen 1998). With these considerations, Ariman (1971), Skalak & Tözeren (1981) and Parvathamma & Devanathan (1983) have used the microstretch and micropolar continuum theories of Eringen to discuss blood flow in arteries.

The ability to deform is crucial to the red blood cell if it is to perform its function of oxygen delivery, and it is also a determinant of cell survival time during circulation (Bor-Kucukatay *et al.* 2005). Nagasawa (1981) and Bor-Kucukatay *et al.* (2005) have measured the elongation index of red blood cells under different conditions. Rogausch (1976) measured the reaction of human red cell deformability on sphering agents.

The problem of axisymmetric Stokesian flow of a micropolar fluid past a sphere with a no-slip boundary condition was discussed by Ramkissoon and Majumdar (1976). Basset (1961) derived the expressions for the force and torque exerted by the fluid on a translating and rotating rigid sphere with a slip-flow boundary condition at its surface (e.g. a settling aerosol sphere). Hoffmann, Marx & Botkin (2007) deduced a formula for the drag acting on the surface of a sphere moving with constant velocity in a micropolar fluid. Gayen & Alam (2006) discussed the algebraic and exponential instabilities in a sheared micropolar granular fluid. The problem of micropolar fluid flows around a sphere and a cylinder was discussed by Hayakawa (2000). Mitarai, Hayakawa & Nakanishi (2002) showed that a micropolar fluid model successfully describes collisional granular flows on a slope. Faltas & Saad (2005) discussed the Stokesian flow with slip caused by the axisymmetric motion of a sphere bisected by a free surface bounding a semi-infinite micropolar fluid. Goldhirsch, Noskowicz & Bar-Lev (2005) derived hydrodynamic equations for nearly smooth granular gases from the pertinent Boltzmann equation. Other micropolar fluid flows are shown in Lukaszewicz (1999). Sherief, Faltas & Saad (2008) discussed the slip at the surface of a sphere translating perpendicular to a plane wall in a micropolar fluid.

Many researchers, as the ones mentioned above, have discussed the micropolar fluid flow problems. However, the microstretch fluid flow has attracted the attention of a low number of researchers. Ieşan (1997) derived a uniqueness theorem for an incompressible microstretch fluid. Narasimhan (2003) considered pulsatile flows of microstretch fluids due to a sinusoidally varying pressure gradient in circular tubes. Ariman (1970) considered the problem of Poiesuille flow between two parallel plates in a microstretch fluid. Eringen (1964) assumed the flow of incompressible microstretch fluids in straight circular arteries to be steady.

Here, we obtain Galerkin-type representations for a microstretch fluid rotating axisymmetrically by using the method of associated matrices, and then we get the fundamental solutions in the cases of concentrated couple and point microstretch force. We then deduce a general formula to evaluate the couple acting on the surface of a rigid body rotating in a microstretch fluid flow. We also apply this result to the problem of a sphere rotating in a microstretch fluid flow with slip and spin boundary conditions. Furthermore, we derive a generalized reciprocal theorem.

2. Field equations

The field equations for an incompressible steady Stokesian microstretch fluid flow are given by (Eringen 1998):

$$\operatorname{div} \boldsymbol{q} = 0, \tag{2.1}$$

$$(\lambda + 2\mu + \kappa)\operatorname{grad}\operatorname{div}\boldsymbol{q} - (\mu+\kappa)\operatorname{curl}\operatorname{curl}\boldsymbol{q} + \kappa\operatorname{curl}\boldsymbol{\nu} - \operatorname{grad} p + \lambda_o \operatorname{grad}\varphi = -\rho\boldsymbol{F}, \tag{2.2}$$

$$(\alpha+\beta+\gamma)\,\text{grad div}\,\boldsymbol{v} - \gamma\,\text{curl curl}\,\boldsymbol{v} + \kappa\,\text{curl}\,\boldsymbol{q} - 2\kappa\,\boldsymbol{v} = -\rho\,C, \tag{2.3}$$

$$a_o(\nabla^2 - \lambda_1)\varphi = -\rho\,L - \pi_o, \tag{2.4}$$

where $\boldsymbol{q}$ and $\boldsymbol{v}$ are the velocity and microrotation vectors; p is the fluid pressure at any point; φ is the microstretch function; ρ is the fluid density; and π_o represents inertial micropressure. Also, $\boldsymbol{F}$, $\boldsymbol{C}$ and L represent body force vector, body couple vector and body microstretch force, respectively. The material constants λ_o, a_o, λ_1, λ, μ and κ represent viscosity coefficients for the translational motions, and α, β and γ are the viscosity coefficients for the rotary motions. The six viscosities λ, μ, κ, α, β and γ are viscosity coefficients for translational and rotational motions of microelements in the case of micropolar fluid flow. Additional viscosities λ_o, a_o and λ_1 over the micropolar fluids emanate from the stretching of the microelements (e.g. suspensions, blood cells, bubbles and polymer melts). All these viscosities are assumed to be constant, depending on the natural state of the fluid. The Laplacian operator is represented by ∇^2.

Neglecting the thermal effect, these material constants have to satisfy the following inequalities (Eringen 1998):

$$\left.\begin{array}{l} 2\mu+\kappa \geqslant 0, \quad \kappa \geqslant 0, \quad \lambda_1 > 0, \quad 3\lambda + 2\mu + \kappa \geqslant 3\lambda_0^2/\lambda_1, \\ \gamma \geqslant 0, \quad \gamma \geqslant |\beta|, \quad 3\alpha+\beta+\gamma \geqslant 0. \end{array}\right\} \tag{2.5}$$

The constitutive equations for the stress tensor, the couple stress tensor, the internal microstretch vector, microstress tensor and the difference between the normal stress T and the micropressure S for an incompressible microstretch fluid flow are, respectively, given by

$$t_{ij} = (\lambda_o\varphi - p)\delta_{ij} + \mu(q_{j,i} + q_{i,j}) + \kappa(q_{j,i} - \varepsilon_{ijk}\boldsymbol{v}_k), \tag{2.6}$$

$$m_{ij} = \alpha\boldsymbol{v}_{r,r}\delta_{ij} + \beta\boldsymbol{v}_{i,j} + \gamma\boldsymbol{v}_{j,i} - b_o\varepsilon_{ijk}\varphi_{,k}, \tag{2.7}$$

$$m_k = a_o\varphi_{,k} + b_o\varepsilon_{kij}\boldsymbol{v}_{i,j}, \tag{2.8}$$

$$S - T = -\pi_o + \lambda_1\varphi, \tag{2.9}$$

where b_o is a material constant and $\boldsymbol{\varepsilon}_{ijk}$ is the alternating tensor defined by

$$\varepsilon_{ijk} = \begin{cases} 1 & \text{if the permutation } (i, j, k) \text{ is even;} \\ -1 & \text{if the permutation } (i, j, k) \text{ is odd;} \\ 0 & \text{otherwise.} \end{cases}$$

3. Galerkin representations of the basic equations

The method of associated matrices utilized by Sandru (1966) and Chowdhury & Glockner (1974) in the theory of elasticity and by Ramkissoon (1977, 1978) in the theory of micropolar fluids is used to obtain Galerkin-type representations. The fundamental singular solutions of the field equations due to a concentrated body couple and a concentrated microstretch force are then obtained.

Our aim now is to solve the system of governing equations (2.1)–(2.4) of eight differential equations of the unknown fields $\boldsymbol{q}$, $\boldsymbol{v}$, p and φ with the aid of the elementary matrix operations. Working with the Cartesian coordinates (x_1, x_2, x_3); we define the Laplacian operator d^2 as follows:

$$\nabla^2 = d^2 = \partial_1^2 + \partial_2^2 + \partial_3^2 \quad \text{with} \quad \partial_i = \frac{\partial}{\partial x_i}, \ i = 1, 2, 3.$$

We shall use the following matrices:

$$\boldsymbol{I}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\quad \boldsymbol{X}=\begin{bmatrix}\partial_1\\ \partial_2\\ \partial_3\end{bmatrix},\quad \boldsymbol{Y}=\begin{bmatrix}0&-\partial_3&\partial_2\\ \partial_3&0&-\partial_1\\ -\partial_2&\partial_1&0\end{bmatrix}\quad\text{and}$$

$$\boldsymbol{Z}=\begin{bmatrix}\partial_1^2&\partial_1\partial_2&\partial_1\partial_3\\ \partial_2\partial_1&\partial_2^2&\partial_2\partial_3\\ \partial_3\partial_1&\partial_3\partial_2&\partial_3^3\end{bmatrix}. \tag{3.1}$$

One can easily verify the following relations:

$$Yu=\nabla\times u,\quad Zu=\nabla(\nabla\cdot u),\quad X^T\boldsymbol{X}=d^2,\quad X^T\boldsymbol{Y}=0,\quad X^T\boldsymbol{Z}=d^2X^T,$$
$$YX=0,\quad YZ=0,\quad Y^2=-d^2\boldsymbol{I}+Z,\quad ZX=d^2X,\quad Z^2=d^2Z,\quad XX^T=Z. \tag{3.2}$$

In the above relations $\boldsymbol{X}^T$denotes the transpose of $\boldsymbol{X}$ and $\boldsymbol{u}=[u_1\quad u_2\quad u_3]^T$ is any vector function.

The system of equations (2.1)–(2.4) can be represented in the following matrix form:

$$\boldsymbol{A}\begin{bmatrix}\boldsymbol{q}\\ \boldsymbol{v}\\ p\\ \varphi\end{bmatrix}=\begin{bmatrix}-\rho\boldsymbol{F}\\ -\rho\boldsymbol{C}\\ 0\\ -\rho L-\pi_o\end{bmatrix}. \tag{3.3}$$

The matrix $\boldsymbol{A}$ is given by

$$\boldsymbol{A}=\begin{bmatrix}L_1\boldsymbol{I}&\kappa\boldsymbol{Y}&-\boldsymbol{X}&\lambda_o\boldsymbol{X}\\ \kappa\boldsymbol{Y}&L_2\boldsymbol{I}+(\alpha+\beta)\boldsymbol{Z}&\boldsymbol{O}&\boldsymbol{O}\\ \boldsymbol{X}^T&\boldsymbol{O}^T&0&0\\ \boldsymbol{O}^T&\boldsymbol{O}^T&0&(a_od^2-\lambda_1)\end{bmatrix}, \tag{3.4}$$

where

$$L_1=(\mu+\kappa)d^2,\quad L_2=\gamma d^2-2\kappa\quad\text{and}\quad O^T=[0\quad 0\quad 0].$$

The solution of the system (3.3) takes the form

$$\begin{bmatrix}\boldsymbol{q}\\ \boldsymbol{v}\\ p\\ \varphi\end{bmatrix}=A^{-1}\begin{bmatrix}-\rho\boldsymbol{F}\\ -\rho\boldsymbol{C}\\ 0\\ -\rho L-\pi_o\end{bmatrix}. \tag{3.5}$$

The problem now reduces to finding the inverse matrix A^{-1} of (3.4). This matrix is found to be

$$A^{-1}=\begin{bmatrix}\dfrac{L_2(d^2\boldsymbol{I}-\boldsymbol{Z})}{L_3d^2}&\dfrac{-\kappa\boldsymbol{Y}}{L_3}&\dfrac{\boldsymbol{X}}{d^2}&\boldsymbol{O}\\ \dfrac{-\kappa\boldsymbol{Y}}{L_3}&\dfrac{L_1L_4\boldsymbol{I}+\{\kappa^2-(\alpha+\beta L_1)\boldsymbol{Z}\}}{L_3L_4}&\boldsymbol{O}&\boldsymbol{O}\\ \dfrac{-\boldsymbol{X}^T}{d^2}&\boldsymbol{O}^T&(\mu+\kappa)&\dfrac{\lambda_o}{(a_od^2-\lambda_1)}\\ \boldsymbol{O}^T&\boldsymbol{O}^T&0&\dfrac{1}{(a_od^2-\lambda_1)}\end{bmatrix}, \tag{3.6}$$

where $L_3 = L_1 L_2 + \kappa^2 d^2$ and $L_4 = (\alpha+\beta+\gamma)d^2 - 2\kappa$. Substituting (3.6) into (3.5), we obtain the following Galerkin-type representations:

$$\begin{aligned} \boldsymbol{q} &= \nabla^2(\gamma\,\nabla^2 - 2\,\kappa)G - (\gamma\,\nabla^2 - 2\kappa)\nabla(\nabla\cdot G) \\ &\quad - \kappa\{(\alpha+\beta+\gamma)\nabla^2 - 2\,\kappa\}\nabla\times\boldsymbol{H} + \nabla\Psi, \end{aligned} \tag{3.7}$$

$$\begin{aligned} \boldsymbol{v} &= -\kappa\,\nabla^2(\nabla\times\boldsymbol{G}) + (\mu+\kappa)\,\nabla^2\{(\alpha+\beta+\gamma)\,\nabla^2 - 2\,\kappa\}\boldsymbol{H} \\ &\quad + \{\kappa^2 - (\mu+\kappa)(\alpha+\beta)\nabla^2\}\nabla(\nabla\cdot\boldsymbol{H}), \end{aligned} \tag{3.8}$$

$$p = -\nabla^2\{\gamma(\mu+\kappa)\nabla^2 - \kappa(2\,\mu+\kappa)\}(\nabla\cdot\boldsymbol{G}) + \lambda_o\Phi, \tag{3.9}$$

$$\varphi = \Phi, \tag{3.10}$$

where the functions $\boldsymbol{G}$, $\boldsymbol{H}$, Ψ and Φ satisfy the following differential equations:

$$\nabla^4\{\gamma(\mu+\kappa)\nabla^2 - \kappa(2\,\mu+\kappa)\}\boldsymbol{G} = -\rho\boldsymbol{F}, \tag{3.11}$$

$$\nabla^2\{\gamma(\mu+\kappa)\nabla^2 - \kappa(2\mu+\kappa)\}\{(\alpha+\beta+\gamma)\nabla^2 - 2\,\kappa\}H = -\rho\boldsymbol{C}, \tag{3.12}$$

$$\nabla^2\Psi = 0, \tag{3.13}$$

$$(a_o\nabla^2 - \lambda_1)\Phi = -\rho\,L - \pi_o. \tag{3.14}$$

4. Three-dimensional concentrated couple

In this section we consider the case of a concentrated couple in the absence of any body force or body microstretch force in an infinite unbound microstretch fluid flow. We take

$$F = 0, \quad \rho C = N\,\delta(x - x_o) \quad \text{and} \quad L = 0, \tag{4.1}$$

where $\boldsymbol{N}$ is a constant vector.

Therefore, the solutions of (3.11)–(3.14) are given by

$$\boldsymbol{H} = \frac{N}{4\pi\,\gamma(\mu+\kappa)(\alpha+\beta+\gamma)r}\left\{\frac{1}{\lambda^2\,\zeta^2} + \frac{\exp(-\lambda\,r)}{\lambda^2(\lambda^2-\zeta^2)} + \frac{\exp(-\zeta\,r)}{\zeta^2(\zeta^2-\lambda^2)}\right\}, \tag{4.2}$$

$$\boldsymbol{G} = 0,\ \Psi = 0 \quad \text{and} \quad \Phi = \frac{\pi_o}{\lambda_1}, \tag{4.3}$$

where

$$\zeta^2 = \frac{2\,\kappa}{(\alpha+\beta+\gamma)}.$$

Substituting the above-obtained results into (3.7)–(3.10), we obtain

$$\begin{aligned} \boldsymbol{v} &= \frac{N}{4\pi\,\gamma}\left[\frac{\exp(-\lambda r)}{r} - \frac{1}{r^3}\left\{\frac{\gamma}{2(2\,\mu+\kappa)} + \frac{(\mu(\alpha+\beta)\zeta^2+\kappa^2)}{\kappa(2\,\mu+\kappa)(\zeta^2-\lambda^2)}(1+\lambda r)\exp(-\lambda r)\right.\right. \\ &\quad \left.\left. + \frac{\gamma}{2\kappa}(1+\zeta r)\exp(-\zeta r)\right\}\right] + \frac{(N\cdot r)\,r}{4\pi\,r^5}\left[\frac{3}{2(2\mu+\kappa)} + \frac{(3+3\,\zeta r+\zeta^2 r^2)}{2\,\kappa}\exp(-\zeta\,\mathrm{r})\right. \\ &\quad \left. + \frac{(\mu(\alpha+\beta)\zeta^2+\kappa^2)}{\gamma\kappa(2\mu+\kappa)(\zeta^2-\lambda^2)}\{3+3\lambda r+\lambda^2 r^2\}\exp(-\lambda r)\right], \end{aligned} \tag{4.4}$$

and

$$\boldsymbol{q} = \frac{1}{4\pi(2\mu+\kappa)}\nabla\times N\left(\frac{1-\exp(-\lambda r)}{r}\right). \tag{4.5}$$

For the case of a point couple of magnitude N_z acting along the z direction, we have $\boldsymbol{N} = N_z\hat{e}_z$, where R, ϕ and z are the cylindrical coordinates. Then the ϕ component of the fluid velocity takes the form

$$q_\phi = \frac{RN_z}{4\pi(2\mu+\kappa)}\left(\frac{1-(1+\lambda r)\exp(-\lambda r)}{r^3}\right). \tag{4.6}$$

If Ω_c represents the angular velocity due to a point couple of magnitude N_z, then

$$\Omega_c = \frac{N_z}{4\pi(2\mu+\kappa)}\left(\frac{1-(1+\lambda r)\exp(-\lambda r)}{r^3}\right). \tag{4.7}$$

At large distances from the obstacle in an unbound medium the flow field becomes identical to the one that would be generated by the action of a point couple equal in magnitude to the couple on the obstacle, provided that the fluid is at rest at infinity; hence

$$\lim_{r\to\infty}\{r^3\,\Omega_c\} = \lim_{r\to\infty}\{r^3\,\Omega\}. \tag{4.8}$$

Therefore, the desired formula can be obtained by taking the limit as $r\to\infty$ for (4.7) to get

$$N_z = 4\pi(2\mu+\kappa)\lim_{r\to\infty}(r^3\Omega), \tag{4.9}$$

where Ω is the angular velocity of the fluid in the case of the presence of the obstacle (rigid body).

In the limiting case when $\kappa\to 0$, the well-known classical result of Stokes for total acting couple in the case of viscous fluids (Lamb 1974) can be recovered, i.e.

$$N_o = 8\pi\mu\lim_{r\to\infty}\{r^3\Omega\}. \tag{4.10}$$

5. Concentrated microstretch force

The case of a concentrated microstretch force density in the absence of any other body force or body couple in an unbound microstretch medium is now considered. For this aim we assume that

$$\boldsymbol{F} = 0, \quad \boldsymbol{C} = 0 \quad \text{and} \quad \rho L = M\delta(x - x_o), \tag{5.1}$$

where M is a constant.

Then, (3.11)–(3.14) have the solutions

$$\boldsymbol{G} = 0, \quad \boldsymbol{H} = 0 \quad \text{and} \quad \Psi = 0, \tag{5.2}$$

$$\Phi = \frac{M}{4\pi a_o}\left(\frac{\exp(-r\sqrt{\lambda_1/a_o})}{r}\right) + \frac{\pi_o}{\lambda_1}. \tag{5.3}$$

Substituting the solutions given above into (3.7)–(3.10), we get

$$\boldsymbol{q} = 0, \quad \boldsymbol{\nu} = 0, \quad \varphi_f = \frac{M}{4\pi a_o}\left(\frac{\exp(-r\sqrt{\lambda_1/a_o})}{r}\right) + \frac{\pi_o}{\lambda_1} \tag{5.4}$$

and the pressure

$$p = \frac{M\lambda_o}{4\pi a_o}\left(\frac{\exp(-r\sqrt{\lambda_1/a_o})}{r}\right) + \frac{\pi_o\lambda_o}{\lambda_1}, \tag{5.5}$$

where φ_f represents the microstretch function due to a microstretch force of magnitude M.

Therefore, the microstretch function due to a point microstretch force of magnitude M acting at the origin is given by

$$M = 4\pi a_o (r \ \exp(-r\sqrt{\lambda_1/a_o})) \left(\varphi_f - \frac{\pi_o}{\lambda_1}\right). \tag{5.6}$$

At sufficiently large distances from the body in an unbound microstretch medium, the flow field becomes identical to the one that would be generated by the action of a point microstretch force equal in magnitude to the microstretch force acting on the body given that the fluid is at rest at infinity; then

$$\lim_{r\to\infty}\left(r \ \exp(-r\sqrt{\lambda_1/a_o})\left\{\varphi_f - \frac{\pi_o}{\lambda_1}\right\}\right) = \lim_{r\to\infty}\left(r \ \exp(-r\sqrt{\lambda_1/a_o})\left\{\varphi - \frac{\pi_o}{\lambda_1}\right\}\right). \tag{5.7}$$

Taking the limit to be r tends to infinity in (5.6), and using the above result we obtain the following relation:

$$M = 4\pi a_o \lim_{r\to\infty}\left(r \ \exp(-r\sqrt{\lambda_1/a_o})\left\{\varphi - \frac{\pi_o}{\lambda_1}\right\}\right). \tag{5.8}$$

6. Generalized reciprocal theorem

In this section we shall prove a reciprocal theorem for the microstretch fluid flow.

Reciprocity theorems have become increasingly important lately because of their uses in the numerical solution of boundary value problems by the boundary element method (BEM; Muldowney & Higdon 1995; Power 1995). This method is rapidly replacing the finite element method in many engineering applications. The BEM needs some theoretical preparations, namely a reciprocity theorem and fundamental solutions, as a starting point.

Theorem:
Let $(q_i, v_i, \varphi, p, \pi_o, S-T, t_{ij}, m_{ij}, m_i, F_i, C_i, L)$ and $(q_i', v_i', \varphi', p', \pi_o', S'-T', t_{ij}', m_{ij}', m_i', F_i', C_i', L')$ be any two motions of the same microstretch fluid, which conform to the equations of motion and the constitutive equations.

Let Σ be a closed surface bounding any fluid volume V and $q_i, v_i, \varphi, q_i', v_i', \varphi' \in C^1$in $\Sigma + V$; then

$$\begin{aligned}&\int_\Sigma [t_{ji}q_i' - t_{ji}'q_i]n_j \, d\Sigma + \int_\Sigma [m_{ji}v_i' - m_{ji}'v_i]n_j \, d\Sigma + \int_\Sigma [m_j\varphi' - m_j'\varphi]n_j \, d\Sigma \\ &\quad + \int_V \rho[F_iq_i' - F_i'q_i]dV + \int_V \rho[C_iv_i' - C_i' v_i]dV + \int_V \rho[L\varphi' - L'\varphi]dV \\ &= \int_V [\pi_o'\varphi - \pi_o\varphi']dV.\end{aligned} \tag{6.1}$$

Proof:
The equations of motion for the two systems have the forms

$$t_{ji,j} + \rho F_i = 0, \tag{6.2}$$
$$m_{ji,j} + \varepsilon_{ijk}t_{jk} + \rho C_i = 0, \tag{6.3}$$
$$m_{j,j} + T - S + \rho L = 0 \tag{6.4}$$

and

$$t'_{ji,j} + \rho F'_i = 0, \tag{6.5}$$

$$m'_{ji,j} + \varepsilon_{ijk} t'_{jk} + \rho C'_i = 0, \tag{6.6}$$

$$m'_{j,j} + T' - S' + \rho L' = 0. \tag{6.7}$$

Multiplying (6.2) by q'_i, (6.3) by ν'_i and (6.4) by φ' and adding and then integrating the sum over V, we obtain by using Gauss' divergence theorem

$$\int_\Sigma [t_{ji} q'_i + m_{ji} \nu'_i + m_j \varphi'] n_j \, \mathrm{d}\Sigma + \int_V \rho [F_i q'_i + C_i \nu'_i + L\varphi'] \mathrm{d}V$$
$$= \int_V [t_{ji} q'_{i,j} + m_{ji} \nu'_{i,j} + m_j \varphi'_{,j} - \varepsilon_{ijk} t_{jk} \nu'_i - (T - S)\varphi'] \mathrm{d}V. \tag{6.8}$$

Interchanging primes, we arrive at the following relation:

$$\int_\Sigma [t'_{ji} q_i + m'_{ji} \nu_i + m'_j \varphi] n_j \, \mathrm{d}\Sigma + \int_V \rho [F'_i q_i + C'_i \nu_i + L'\varphi] \mathrm{d}V$$
$$= \int_V [t'_{ji} q_{i,j} + m'_{ji} \nu_{i,j} + m'_j \varphi_{,j} - \varepsilon_{ijk} t'_{jk} \nu_i - (T' - S') \varphi] \mathrm{d}V. \tag{6.9}$$

Using (2.6)–(2.9) with the aid of the relation

$$\varepsilon_{ikn} \cdot \varepsilon_{nmj} = \delta_{im} \delta_{kj} - \delta_{ij} \delta_{km},$$

we get

$$t_{ji} q'_{i,j} + m_{ji} \nu'_{i,j} + m_j \varphi'_{,j} - (T - S)\varphi' - \varepsilon_{ijk} t_{jk} \nu'_i + \pi_o \varphi'$$
$$= t'_{ji} q_{i,j} + m'_{ji} \nu_{i,j} + m'_j \varphi_{,j} - (T' - S')\varphi - \varepsilon_{ijk} t'_{jk} \nu_i + \pi'_o \varphi. \tag{6.10}$$

Subtracting (6.8) from (6.9) and using (6.10) we obtain the required formula.

In the absence of the microstretch force, the relation (6.1) reduces to

$$\int_\Sigma [t_{ji} q'_i - t'_{ji} q_i] n_j \, \mathrm{d}\Sigma + \int_\Sigma [m_{ji} \nu'_i - m'_{ji} \nu_i] n_j \, \mathrm{d}\Sigma$$
$$+ \int_V \rho [F_i q'_i - F'_i q_i] \mathrm{d}V + \int_V \rho [C_i \nu'_i - C'_i \nu_i] \mathrm{d}V = 0, \tag{6.11}$$

a result obtained by Ramkissoon (1975) for a micropolar fluid.

7. Axisymmetric rotation of a rigid sphere in an unbound microstretch fluid with slip and spin

As an application, we consider the axially symmetric slow, steady rotation of a sphere, of radius a, with a constant angular velocity Ω_o in an unbound microstretch fluid flow. It is appropriate to use the spherical polar coordinates (r, θ, ϕ). The velocity components of the rigid sphere in these coordinates are given by

$$V_r = 0, \quad V_\theta = 0 \quad \text{and} \quad V_\phi = \Omega_o a \sin\theta.$$

Thus, the velocity and the microrotation vectors have the following components:

$$\boldsymbol{q} = (0, 0, q_\phi(r, \theta)), \quad \boldsymbol{\nu} = (\nu_r(r, \theta), \nu_\theta(r, \theta), 0). \tag{7.1}$$

The following boundary conditions are satisfied on the surface of the sphere:

$$\boldsymbol{v}\Big|_{boundary} = \frac{n}{2}\text{curl}q\Big|_{boundary} \quad \text{and} \quad \varphi = 0 \quad \text{on } r = a, \tag{7.2}$$

where n is the spin parameter that varies from 0 to 1. This parameter is assumed to depend only on the nature of the fluid and solid boundary.

The slip boundary condition satisfied on the surface of the sphere takes the form

$$\beta_1(q_\phi - V_\phi) = t_{r\phi} \quad \text{on} \quad r = a, \tag{7.3}$$

where β_1 is termed as sliding friction coefficient. This coefficient is a measure of the degree of tangential slip existing between the fluid and the solid boundary. It is assumed to depend only on the nature of the fluid and solid surface (Basset 1961; Faltas & Saad 2005). In the limiting case, when $\beta_1 \rightarrow \infty$, we return to the classical case of no-slip boundary condition.

Taking the substitutions

$$P(r,\theta) = \text{div}\,\boldsymbol{v}, \quad Q(r,\theta)\hat{e}_\phi = \text{curl}\,\boldsymbol{v} \tag{7.4}$$

and using the non-dimensional variables

$$q_\phi^* = \frac{q_\phi}{a\Omega_o}, \quad \boldsymbol{v}_i^* = \frac{a^2\kappa}{\gamma\Omega_o}\boldsymbol{v}_i, \quad r^* = \frac{r}{a}, \quad \varphi^* = \frac{a_o}{\pi_o a^2}\varphi, \quad \Omega^* = \frac{\Omega}{\Omega_o},$$
$$t_{ij}^* = \frac{a^2}{\gamma\Omega_o}t_{ij}, \quad m_{ij}^* = \frac{\kappa a^3}{\gamma^2\Omega_o}m_{ij}, \quad m_k^* = \frac{m_k}{\pi_o a}, \quad N_z^* = \frac{N_z}{\gamma\Omega_o a}, \tag{7.5}$$

the governing equations (2.2)–(2.4) take the form (dropping asterisks for convenience)

$$\{\nabla^2 - N^2\}P = 0, \tag{7.6}$$
$$\{L_{-1} - \hat{\lambda}^2\}Q = 0, \tag{7.7}$$
$$L_{-1}\{L_{-1} - \hat{\lambda}^2\}q_\phi = 0, \tag{7.8}$$
$$\{\nabla^2 - \ell^2\}\varphi = 0, \tag{7.9}$$

where

$$N^2 = \frac{2\kappa a^2}{(\alpha+\beta+\gamma)}, \quad \hat{\lambda}^2 = \frac{(2\mu+\kappa)\kappa a^2}{(\mu+\kappa)\gamma} \quad \text{and} \quad \ell^2 = \frac{\lambda_1 a^2}{a_o}.$$

Moreover, ∇^2 is the Laplacian operator and the generalized axisymmetric Stokesian operator L_{-1} is defined by

$$L_{-1} = \nabla^2 - \frac{1}{r^2\sin^2\theta}.$$

Solving (7.6) and (7.8) and retaining only the bound terms, we obtain the non-dimensional velocity component in the form

$$q_\phi = \left\{\frac{A'}{r^2} + \frac{B'}{\sqrt{r}}K_{3/2}(\hat{\lambda}r)\right\}\sin\theta. \tag{7.10}$$

It should be noted here that (7.10) represents the non-dimensional (scaled) velocity component. In order to obtain the physical velocity component we use the non-dimensional variables of the velocity q_ϕ and the distance r given in (7.5) to get

$$q_\phi = a^3\Omega_o\left\{\frac{A'}{r^2} + \frac{B'}{\sqrt{r}}K_{3/2}(\hat{\lambda}r)\right\}\sin\theta.$$

As a special case for a viscous fluid with no-slip boundary condition ($\kappa \to 0$, $\beta_1 \to \infty$), we have $q_\phi = a^3 \Omega_o / r^2 \sin\theta$.

Also, we have

$$P = \frac{C'}{\sqrt{r}} K_{3/2}(Nr)\cos\theta, \tag{7.11}$$

$$Q = -\frac{a^2\hat{\lambda}^2(\mu+\kappa)\,B'}{\gamma\sqrt{r}} K_{3/2}(\hat{\lambda}r)\sin\theta, \tag{7.12}$$

where $K_\nu(.)$ denotes the modified Bessel function of second kind of the order $\boldsymbol{\nu}$ and A', B' and C' are constants to be determined from the boundary conditions.

Thus, the microrotation components take the forms

$$\nu_r = \left\{ \frac{\kappa a^2 A'}{\gamma r^3} + \frac{2(\mu+\kappa)a^2 B'}{\gamma r^{3/2}} K_{3/2}(\hat{\lambda}r) - \frac{C'}{N^2 r^{3/2}}(2\,K_{3/2}(Nr) + Nr\,K_{1/2}(Nr)) \right\}\cos\theta, \tag{7.13}$$

$$\nu_\theta = \left\{ \frac{\kappa a^2 A'}{2\gamma r^3} + \frac{(\mu+\kappa)a^2 B'}{\gamma\, r^{3/2}} (K_{3/2}(\hat{\lambda}r) + \hat{\lambda}r\,K_{1/2}(\hat{\lambda}r)) - \frac{C'}{N^2 r^{3/2}} K_{3/2}(Nr) \right\}\sin\theta. \tag{7.14}$$

Applying the boundary conditions (7.2) and (7.3), in non-dimensional form, we obtain a system of linear algebraic equations in the unknown variables A', B' and C', whose solutions give

$$\begin{aligned} A' = \frac{2}{A_o}&\{n\kappa\Gamma[\gamma N^2(2\mu+\kappa)(\hat{\lambda}+1) + (\gamma\hat{\lambda}^2(\mu+\kappa) + \{(\mu+\kappa)+\gamma(8\mu+5\kappa)\} \\ &\times(\hat{\lambda}+1))(N+1)] + (2\mu+\kappa)[n\kappa\gamma(\hat{\lambda}+1)(N^2+3N+3) \\ &+(\mu+\kappa)(N^2(\gamma\,\hat{\lambda}^2+\hat{\lambda}+1) + 2(N+1)\{\gamma\hat{\lambda}^2 + (1-\gamma)\hat{\lambda} + 1 - \gamma\})]\}, \end{aligned} \tag{7.15}$$

$$\begin{aligned} B' = \frac{-\gamma\kappa\hat{\lambda}^{3/2}e^{\hat{\lambda}}}{A_o\sqrt{\pi/2}}&\{3n\,\Gamma[(2\mu+\kappa)N^2 + 2(3\mu+2\kappa)(N+1)] \\ &+(2\mu+\kappa)[(2n+1)N^2 + 6n(N+1)]\}, \end{aligned} \tag{7.16}$$

$$\begin{aligned} C' = \frac{-\kappa a(2\mu+\kappa)N^{7/2}e^N}{\gamma A_o\sqrt{\pi/2}}&\{3n\Gamma[\gamma\hat{\lambda}^2(\mu+\kappa) + \{(\mu+\kappa)+\gamma(2\mu+\kappa)\}(\hat{\lambda}+1)] \\ &+2\gamma\hat{\lambda}^2(\mu+\kappa)(n-1) + n\,(2(\mu+\kappa)+\gamma(4\mu+\kappa)) \\ &\times(\hat{\lambda}+1) - 2(\mu+\kappa)(1-\gamma)(\hat{\lambda}+1)\}, \end{aligned} \tag{7.17}$$

where

$$\begin{aligned} A_o = (2\mu+\kappa)\{&\Gamma[N^2\{3\gamma\,\hat{\lambda}^2(\mu+\kappa) + (3(\mu+\kappa)-\gamma\kappa)(\hat{\lambda}+1)\} + 4(\mu+\kappa) \\ &\times\{\gamma\,\hat{\lambda}^2 + (1-\gamma)(\hat{\lambda}+1)\}(N+1)] + 2\gamma\,\hat{\lambda}^2(\mu+\kappa)(N^2+2N+2) \\ &+[N^2\{2(\mu+\kappa)-\gamma\kappa\} + 4(\mu+\kappa)(1-\gamma)(N+1)](\hat{\lambda}+1)\} \end{aligned}$$

and

$$\Gamma = \frac{2\mu+\kappa}{a\beta_1}.$$

The microstretch function φ can be obtained, by solving (7.9) and then applying the second of the boundary conditions (7.2), as follows:

$$\varphi = \frac{1}{\ell^2}\left\{1 - \frac{K_{1/2}(\ell r)}{\sqrt{r}K_{1/2}(\ell)}\right\}. \tag{7.18}$$

The non-vanishing components of the stress tensor are given by

$$t_{rr} = t_{\theta\theta} = t_{\phi\phi} = \frac{\tau_1'}{\ell^2}, \tag{7.19}$$

$$t_{r\phi} = -\left\{\frac{(2\mu+\kappa)a^2}{\gamma}\left[\frac{3A'}{2r^3} + \frac{B'}{r^{3/2}}K_{3/2}(\hat{\lambda}r)\right] + \frac{C'}{N^2r^{3/2}}K_{3/2}(Nr)\right\}\sin\theta, \tag{7.20}$$

$$t_{\phi r} = -\left\{\frac{(2\mu+\kappa)a^2}{\gamma}\left[\frac{3A'}{2r^3} + \frac{B'}{r^{3/2}}(2K_{3/2}(\hat{\lambda}r) + \hat{\lambda}\,r K_{1/2}(\hat{\lambda}r))\right] - \frac{C'}{N^2r^{3/2}}K_{3/2}(Nr)\right\}\sin\theta, \tag{7.21}$$

$$t_{\theta\phi} = -\left\{\frac{(2\mu+\kappa)a^2B'}{\gamma r^{3/2}}K_{3/2}(\hat{\lambda}r) + \frac{C'}{N^2r^{3/2}}[2K_{3/2}(Nr) + Nr\,K_{1/2}(Nr)]\right\}\cos\theta = -t_{\phi\theta}. \tag{7.22}$$

Also, the difference between the normal stress and the micropressure is given by

$$T - S = \left\{\frac{\pi_o a^3}{\gamma V}\right\}\frac{\exp(\ell - \ell r)}{r}. \tag{7.23}$$

The couple stress tensor has the following components:

$$m_{rr} = \frac{(\beta+\gamma)}{\gamma}\left\{\frac{-3\kappa a^2A'}{\gamma r^4} - \frac{2(\mu+\kappa)a^2B'}{\gamma r^{5/2}}[3K_{3/2}(\hat{\lambda}r) + \hat{\lambda}\,r K_{1/2}(\hat{\lambda}r)]\right.$$
$$\left. + \frac{C'}{N^2r^{5/2}}\left[\left(6 + \frac{(\alpha+\beta+\gamma)N^2r^2}{(\beta+\gamma)}\right)K_{3/2}(Nr) + 2Nr\,K_{1/2}(\hat{\lambda}r)\right]\right\}\cos\theta, \tag{7.24}$$

$$m_{\theta\theta} = \frac{(\beta+\gamma)}{\gamma}\left\{\frac{3\kappa a^2A'}{2\gamma r^4} + \frac{(\mu+\kappa)a^2B'}{\gamma r^{5/2}}[3K_{3/2}(\hat{\lambda}r) + \hat{\lambda}\,r K_{1/2}(\hat{\lambda}r)]\right.$$
$$\left. - \frac{C'}{N^2r^{5/2}}\left[\left(3 - \frac{\alpha N^2r^2}{(\beta+\gamma)}\right)K_{3/2}(Nr) + N\,r\,K_{1/2}(\hat{\lambda}r)\right]\right\}\cos\theta, \tag{7.25}$$

$$m_{\phi\phi} = \frac{\alpha C'}{\gamma\sqrt{r}}K_{3/2}(Nr)\cos\theta, \tag{7.26}$$

$$m_{r\theta} = \left\{\frac{-3\kappa a^2(\beta+\gamma)A'}{2\gamma^2r^4} - \frac{(\mu+\kappa)a^2B'}{\gamma r^{5/2}}\left[\left(\frac{3(\beta+2\gamma)}{\gamma} + \hat{\lambda}^2r^2\right)K_{3/2}(\hat{\lambda}r)\right.\right.$$
$$\left.\left. + \frac{(\beta+\gamma)\hat{\lambda}\,r}{\gamma}K_{1/2}(\hat{\lambda}r)\right] + \frac{C'}{N^2r^{5/2}}\left[\frac{3\beta}{\gamma}K_{3/2}(Nr) + \frac{(\beta-\gamma)Nr}{\gamma}K_{1/2}(\hat{\lambda}r)\right]\right\}\sin\theta, \tag{7.27}$$

$$m_{\theta r} = \left\{\frac{-3\kappa a^2(\beta+\gamma)A'}{2\gamma^2r^4} - \frac{(\mu+\kappa)a^2B'}{\gamma r^{5/2}}\left[\left(\frac{3(2\beta+\gamma) + \beta\,\hat{\lambda}^2r^2}{\gamma}\right)K_{3/2}(\hat{\lambda}r)\right.\right.$$
$$\left.\left. + \frac{(\beta+\gamma)\hat{\lambda}r}{\gamma}K_{1/2}(\hat{\lambda}r)\right] + \frac{C'}{N^2r^{5/2}}\left[3K_{3/2}(Nr) - \frac{(\beta-\gamma)Nr}{\gamma}K_{1/2}(\hat{\lambda}r)\right]\right\}\sin\theta, \tag{7.28}$$

$$m_{\phi\theta} = -m_{\theta\phi} = \frac{\tau_2'K_{3/2}(\ell r)}{\ell\sqrt{r}K_{1/2}(\ell)}. \tag{7.29}$$

The non-vanishing components of the microstretch vector of the problem at hand also take the following forms:

$$m_r = \frac{K_{3/2}(\ell r)}{\ell\sqrt{r}K_{1/2}(\ell)}, \tag{7.30}$$

$$m_\phi = \tau_3'\left\{\frac{(\mu+\kappa)a^2B'}{\gamma r^{5/2}}(3+\hat{\lambda}^2r^2)K_{3/2}(\hat{\lambda}r) + \frac{C'}{N^2r^{5/2}}[3K_{3/2}(Nr)] + 2NrK_{1/2}(Nr)\right\}\sin\theta, \tag{7.31}$$

where

$$\tau_1' = \frac{\lambda_o\pi_o a^4}{a_o\gamma\,\Omega}, \quad \tau_2' = \frac{b_o\pi_o\kappa a^4}{a_o\gamma^2\Omega} \quad \text{and} \quad \tau_3' = \frac{b_o\gamma\,\Omega}{\pi_o\kappa a^4}.$$

The total couple acting on the surface of the rigid sphere, in non-dimensional form, is given by

$$N_z = N_z^s + N_z^c,$$

where N_z^s and N_z^c are the total acting couples due to the stress and the couple stress components, in non-dimensional form, respectively.

We now have

$$N_z^s = -\int_S (r\times(\hat{n}:t))\cdot\hat{k}\,dS, \tag{7.32}$$

where $r=\hat{e}_r$, $(\hat{n}:t)=t_{rr}\hat{e}_r+t_{r\theta}\hat{e}_\theta+t_{r\phi}\hat{e}_\phi$ and $\hat{k}=\cos\theta\,\hat{e}_r-\sin\theta\,\hat{e}_\theta$, $(\hat{e}_r,\hat{e}_\theta,\hat{e}_\phi)$ represent the unit vectors along the directions of r, θ, ϕ, respectively, and $\hat{n}$and $\hat{k}$are, respectively, the unit normal vector outward from the surface of the sphere and the unit vector along z-axis.

Therefore, (7.32) takes the form

$$N_z^s = -2\pi\int_0^\pi t_{r\phi}\sin^2\theta\,\mathrm{d}\theta. \tag{7.33}$$

Substituting for $t_{r\phi}$ and then evaluating the resulting integral, we obtain

$$N_z^s = \frac{8\pi}{3}\left[\frac{(2\mu+\kappa)a^2}{\gamma}\left\{\frac{3}{2}A' + B'K_{3/2}(\hat{\lambda})\right\} + \frac{C'}{N^2}K_{3/2}(N)\right]. \tag{7.34}$$

We also have

$$N_z^c = -\int_S (\hat{n}:m))\cdot\hat{k}\,\mathrm{d}S, \tag{7.35}$$

where$(\hat{n}:m)=m_{rr}\hat{e}_r+m_{r\theta}\hat{e}_\theta+m_{r\phi}\hat{e}_\phi$.

Using the non-dimensional variables (7.5), (7.35) takes the form

$$N_z^c = \frac{-2\pi\gamma}{\kappa a^2}\int_0^\pi (m_{rr}\cos\theta - m_{r\theta}\sin\theta)\sin\theta\,\mathrm{d}\theta, \tag{7.36}$$

which can be simplified as

$$N_z^c = \frac{-8\pi}{3}\left[\frac{(2\mu+\kappa)a^2}{\gamma}B'K_{3/2}(\hat{\lambda}) + \frac{C'}{N^2}K_{3/2}(N)\right]. \tag{7.37}$$

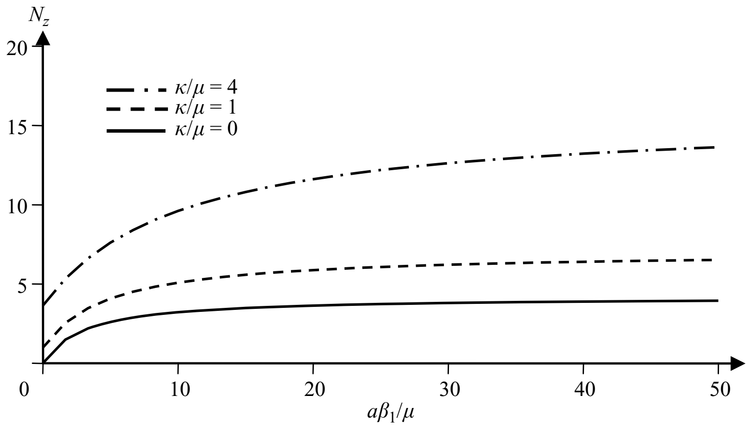

FIGURE 1. Total couple distribution for $n = 1$.

Adding (7.34) and (7.37), we obtain the total non-dimensional couple acting on the surface of the rigid sphere in the following form:

$$N_z = \frac{4\pi(2\mu + \kappa)a^2}{\gamma} A'. \tag{7.38}$$

As an application of our main result (4.9), we now obtain the couple on a rotating sphere. From (7.10), the non-dimensional angular velocity of the fluid flow takes the form

$$\Omega = \left\{ \frac{A'}{r^3} + \frac{B'}{r\sqrt{r}} K_{3/2}(\hat{\lambda} r) \right\}. \tag{7.39}$$

Applying the non-dimensional variables (7.5) to the relation (4.9), we can easily obtain the non-dimensional couple experienced by the sphere as

$$N_z = 4\pi \frac{(2\mu + \kappa)a^2}{\gamma} \lim_{r\to\infty} (r^3 \Omega) \tag{7.40}$$

$$= \frac{4\pi(2\mu + \kappa)a^2}{\gamma} A', \tag{7.41}$$

which is identical to (7.38).

8. Numerical results

In the present section we present the resultant couple acting on the surface of the sphere, the velocity and the microrotation components graphically. In figure 1, the variation of the total acting couple on the surface of the rigid sphere against the sliding coefficient $\beta_1 a/\mu$ for different values of micropolarity coefficient κ/μ when the spin parameter $n = 1$ is represented. Figure 2 shows the total couple against the sliding coefficient for different values of the parameter n when $\kappa/\mu = 2$. Figures 3–5 represent the distributions of the velocity and microrotation components for a constant value of θ, respectively, against r for different values of the sliding coefficient $\beta_1 a/\mu$ when $\kappa/\mu = 1$ and $n = 0.1$. Figure 6 shows the variation of the microstretch against r for different values of the parameter ℓ.

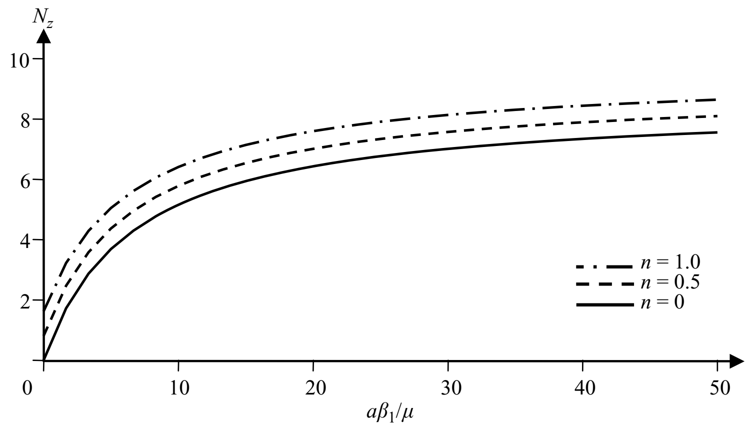

FIGURE 2. Total couple distribution for $\kappa/\mu = 2$.

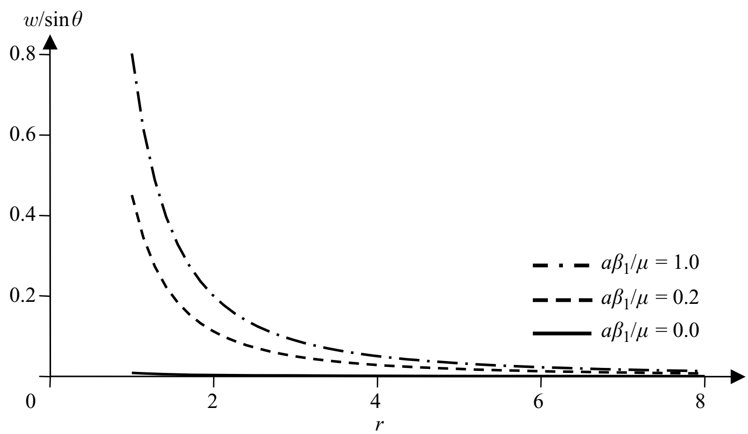

FIGURE 3. Velocity distribution.

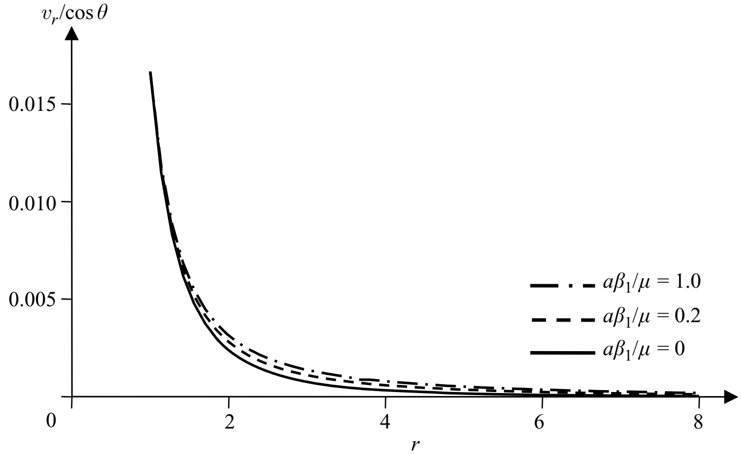

FIGURE 4. Microrotation distribution.

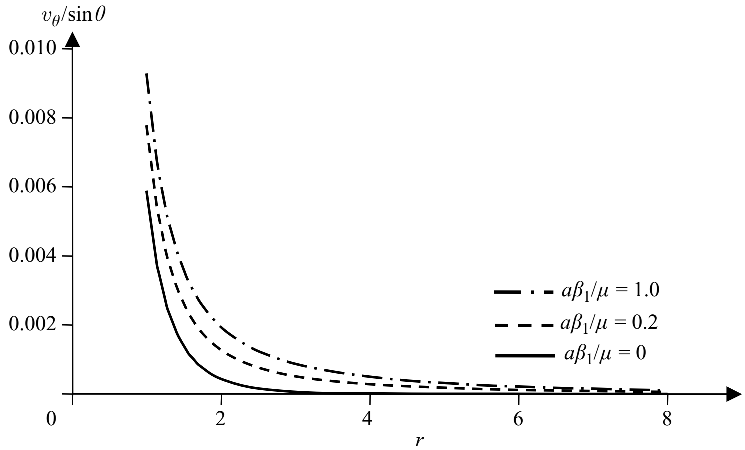

FIGURE 5. Microrotation distribution.

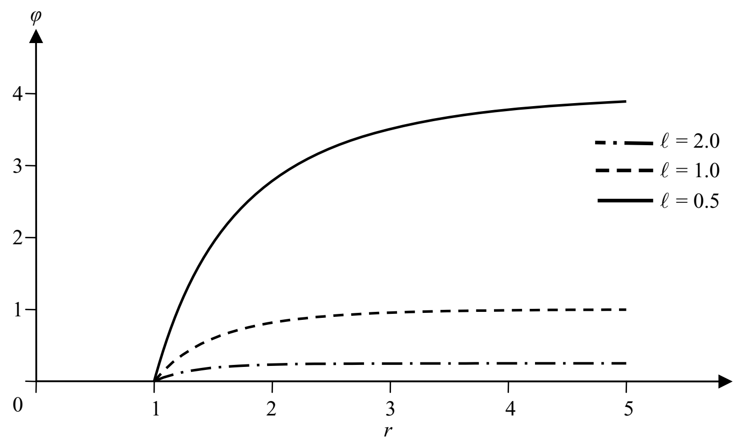

FIGURE 6. Microstretch function distribution.

9. Conclusion

The following are the conclusions of this paper:

(1) From the obtained formula (4.9), we note that the total couple acting on a rigid body rotating axisymmetrically in a microstretch fluid has the same value as in the case of a micropolar fluid. This means that the microstretch force does not contribute to the resultant couple acting on the surface of the rigid body.

(2) From (4.9) and (4.10), it can be observed that the total acting couple on the rigid body in a microstretch fluid flow cannot exceed $(1+\kappa/\mu)$ times the total couple in the case of Newtonian fluid flow. Also, its minimum value is greater than $(1+\kappa/2\mu)$ times N_o.

(3) The magnitude of the total couple shown in figure 1 is directly proportional to the value of the slip parameter β_1. As a limiting case as $\beta_1 \to 0$, we recover the case of perfect slip, while the classical case of no slip can be obtained as $\beta_1 \to \infty$.

(4) From figure 2 it is observed that the magnitude of the total couple increases with the increase of the value of either the spin coefficient n or the micropolarity coefficient κ.

(5) For a fixed value of θ, each of the velocity components and the microrotation components, represented in figures 3–5, varies inversely with r and directly with β_1.

(6) From figure 6, it can be shown that near the sphere the values of the microstretch φ increase with the increase of r, and away from the sphere the microstretch φ tends to be a constant. It also increases monotonically with the increase of the parameter ℓ.

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